Purdue University
Purdue e-Pubs

International Compressor Engineering Conference

School of Mechanical Engineering

1992

Application of the Economic Elasticity Concept to Compressor Performance Parameters

K. W. Yun
United Technologies Carrier Corporation

APPLICATION OF THE ECONOMIC ELASTICITY CONCEPT TO COMPRESSOR PERFORMANCE PARAMETERS

K.W. Yun
United Technologies Carrier Corp.
Syracuse, New York

ABSTRACT

Introducing the economic concept of elasticity, elasticity coefficients involving performance variables of compressors can be defined. Such a coefficient compares the percentage change in one variable with the percentage change in another. Using the published performance of three types of similar capacity compressors a total of twenty elasticity coefficients is computed. The computed coefficients are then classified into three general groups. Two specific coefficients, capacity elasticity of power and power elasticity of EER, are selected for further analysis. Also discussed is the former coefficient as a function of compression ratio. Finally, compressor run time, type of compressor, operating condition and compression ratio are cited as factors affecting elasticity values.

INTRODUCTION

Performance variables or parameters of a compressor include capacity, input power, energy efficiency ratio. And design, manufacturing and quality factors determine their values. Knowing qualitative interrelationship among these variables is not good enough in checking compressor performance. Given a set of performance values, a quantitative analysis of the interrelationship among the parameters is a prerequisite for performance improvements. In this effort, an analytical technique in economics is introduced. Also shown are results of applying the economic concept to various types of compressor and operating conditions.

THE ECONOMIC CONCEPT OF ELASTICITY

In economic studies, demand or supply curves which relate price to quantity demand or supplied are of a significant importance. Related to these curves, the economic concept of elasticity is widely used to express the responsiveness of quantity demand to price changes. Price elasticity coefficient of demand (supply) expresses the responsiveness. The coefficient compares the percentage change in quantity demanded with the percentage change in price.

Numerically, a typical demand curve passes through all three ranges, $|E| < 1$, $|E| = 0$, $|E| > 1$. Elastic (inelastic) demand means that over a given portion of the demand curve, a lower price would result in a proportionately larger (smaller) increase in quantity and increased (decreased) revenue. Unit elasticity means that a change in price would result in proportionate change in quantity with no change in revenue or total expenditure.

MATHEMATICAL EXPRESSION OF THE ELASTICITY CONCEPT

Mathematically, the concept bases the idea on the ratio of the percentage change in one variable to the percentage change in another. The general definition of Y elasticity of X, Ey, is percentage change in X divided by percentage change in Y.

To express the elasticity, we start with the expression of percentage change in variable X:

$$Percent\ change\ in\ X = \frac{Amount\ changed\ in\ X}{Average\ of\ X_1\ and\ X_2} = \frac{\Delta X}{X_0} = \frac{X_2 - X_1}{\frac{X_1 + X_2}{2}} \qquad (1)$$

where X_1 represents the X before the change, X_2 after the change, and X_0 being average of X_1 and X_2. Note the convention of using the midpoint of the variable range as a basis for calculating the percentage.

Likewise, using the same convention as above:

$$Percentage\ change\ in\ Y = \frac{Amount\ Changed\ in\ Y}{Average\ of\ Y_1\ and\ Y_2} = \frac{\Delta Y}{Y_0} = \frac{Y_2 - Y_1}{\frac{Y_1 + Y_2}{2}} \qquad (2)$$

The Y Elasticity of X is then:

$$E_y = \frac{Percentage\ change\ in\ X}{Percentage\ change\ in\ Y} = \frac{(1)}{(2)} = \frac{\frac{\Delta X}{X_0}}{\frac{\Delta Y}{Y_0}} = \frac{\Delta X}{\Delta Y}\frac{Y_0}{X_0} \qquad (3)$$

The above formula is an arc elasticity. As ΔY approaches to 0, the arc elasticity becomes point elasticity. The definition of point elasticity then becomes:

$$E_y = \frac{dX}{dY}\frac{Y_0}{X_0} \qquad (4)$$

where $[\Delta X/\Delta Y]$ transforms to $[dX/dY]$ as ΔY approaches to 0 in the limit.

CHARACTERISTICS OF THE ELASTICITY

There are several important characteristics to note in the elasticity formula:

(i) Elasticity is a relative measure, and the values range from minus infinity to plus infinity. Though it is customary in economic analysis to drop the negative sign, in our engineering analysis we shall not do so as the $\pm$ sign carries an important significance.

(ii) A positively [negatively] sloped curve yields a positive [negative] value for elasticity.

(iii) The formula (3) for computing elasticity consists of two parts - one representing slope $\Delta X/\Delta Y$ and the other position; the point (Y_0/X_0)at which the measurement is being taken. Thus one cannot look at a curve and determine its elasticity at a given point without referring to Y and X axes. This is because both the slope and position of a curve determine elasticity at any given point on the curve.

THE IMPORTANCE OF ELASTICITY

If elasticity is nothing more than another way of describing the shape and position of demand curves, it would not deserve much attention. What makes elasticity an important concept is not so much its descriptive usefulness as its analytic usefulness. The importance of elasticity comes from three factors: magnitude, direction (± sign) of elasticity value, and magnitude of the resultant parameter from the product of the two variables. The sections following discuss significance of each of these three factors.

CALCULATION OF ELASTICITY COEFFICIENTS

We shall apply the concept of elasticity to compressor performance to different types of compressors - rotary-type compressor of one manufacturer, and reciprocating as well as scroll-type of the other manufacture. Published data from three types of compressors, with an approximate capacity of 24,000 btuh (6,050 kcal/hr) are used to calculate elasticity coefficients.

Computation of different elasticity coefficients uses a total of five performance parameters. Selected performance variables are EER in addition to capacity, input power, mass flow rate and amperage which compressor manufacturers publish. Out of 20 different possible elasticities, one half of them are just inverse of another. Furthermore, close intra-parameter relationship (such as capacity and mass flow rate) result in basically same values of elasticity. The following table summarizes classification of coefficients in three arbitrary ranges:

Y ELASTICITY OF X

\ X Y \	X=Capacity	Power	EER	Mass Flow	Amperage
Y=Capacity	-	I	U	U	I
Power	S	-	S	S	U
EER	U	I	-	U	I
Mass Flow Rate	U	I	U	-	I
Amperage	S	U	S	S	-

where I: Insensitive (Inelastic) $|E| \approx 0 - 0.5$

U: Unitary $|E| \approx 0.5 - 1.5$

S: Elastic (Sensitive) $|E| \approx 0 - 0.5$

Among the above elasticity, we shall examine the following two elasticity coefficients for further analysis.

CAPACITY ELASTICITY OF POWER INPUT

When we wish to express a relationship between two variables, such as compressor capacity and power input, functionally we can show how one variable changes with movements in the other. How sensitively power changes as compressor capacity changes is one of our interests as it relates to compressor energy efficiency ratio. Thus, we are interested in the characteristics of the capacity elasticity of power input.

We define the elasticity as the percentage change in power input as a result of percentage change in compressor capacity. Denoting capacity elasticity of power as Ec:

where W and C stand for input power and compressor capacity respectively. Input power is elastic when the percentage change in input power is greater than the percentage change in

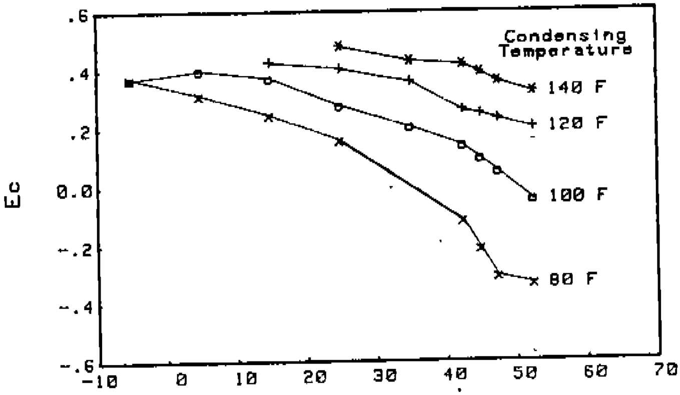
$$E_c = \frac{\text{Percentage change in power input}}{\text{Percentage change in capacity}} = \frac{\frac{\Delta W}{W}}{\frac{\Delta C}{C}} = \frac{\Delta W}{\Delta C}\frac{C}{W} \qquad (5)$$

capacity i.e., when $\Delta W/W$ is greater than $\Delta C/C$, in elastic when $\Delta W/W$ is less than $\Delta C/C$, and unitary when $\Delta W/W$ is equal to $\Delta C/C$.

The elasticity varies with operating condition as one might suspect. For a given (reciprocating) type of compressor, values of Ec are plotted (Fig 1) over the compressor operating range.

Capacity elasticity of input power is inelastic over the wide operating range. The elasticity decreases with increasing evaporating temperature (Te) and approaches to zero at near 50°F (10°C). This observation for certain compressors relates to the decreasing slope of input power curves with Te. As Te decreases the coefficient converges to a certain value (0.4 in this case). This observation is not obvious from data or performance curves; one must compute the coefficient to find this type of characteristic.

Fig.1 Capacity Elasticity (Ec) of Input Power



When the different types of compressors are compared, there are differences. Despite the differences among them, they are still all inelastic. However, the plus/minus sign of the values is significant as it says that input power varies in the opposite direction from capacity change.

POWER ELASTICITY OF ENERGY EFFICIENCY RATIO

We define power elasticity of EER as the percentage change in EER as a result of percentage change in input power. Denoting elasticity of efficiency as Ew:

$$E_w = \frac{Percentage\ change\ in\ EER}{Percentage\ change\ in\ W} = \frac{\frac{\Delta R}{R}}{\frac{\Delta W}{W}} = \frac{\Delta R}{\Delta W}\frac{W}{R} \qquad (6)$$

where R and W stand for EER and input power respectively.

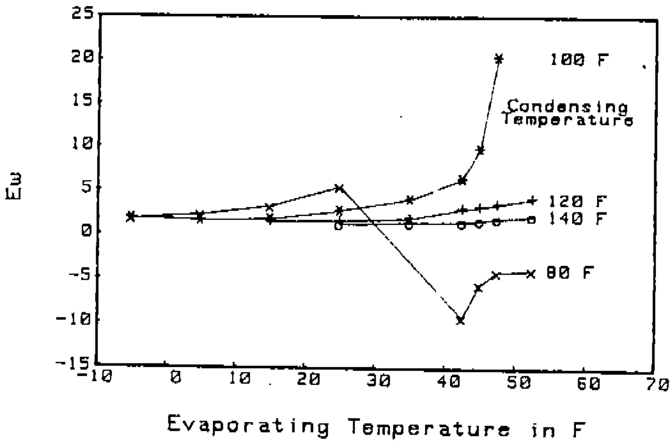
Efficiency is elastic when the percentage change in EER is greater than the percentage change in input power - i.e. when $\Delta R/R$ is greater than $\Delta W/W$, inelastic when $\Delta R/R$ is less than $\Delta W/W$, and unitary when $\Delta R/R$ is equal to $\Delta W/W$.

Unlike the capacity elasticity of input power (Ec), the power elasticity of EER (Ew) provides a more useful analysis. Capacity being calculated as the power input multiplied by EER, a change in power can have one of three effects on capacity, depending on the shape and position of the curve. If EER changes by a larger (smaller) (same) percentage than power input, so that capacity increases (decreases) (remains unchanged) as the power decreases, EER is said to be elastic (inelastic) (unit elastic).

The fact that capacity is the product of power and EER explains why capacity responds in such a fashion to power changes. Which way the product of these two, that is, capacity, goes depends upon which changes the most (figured as a percentage). Compressor will lose capacity by a power if EER (demand) is relatively inelastic, higher EER for lower capacity. It tells something about capacity. Whether power elasticity of EER for a specific compressor is elastic or inelastic is very important because resultant net capacity change of increasing or decreasing power hinges on the elasticity.

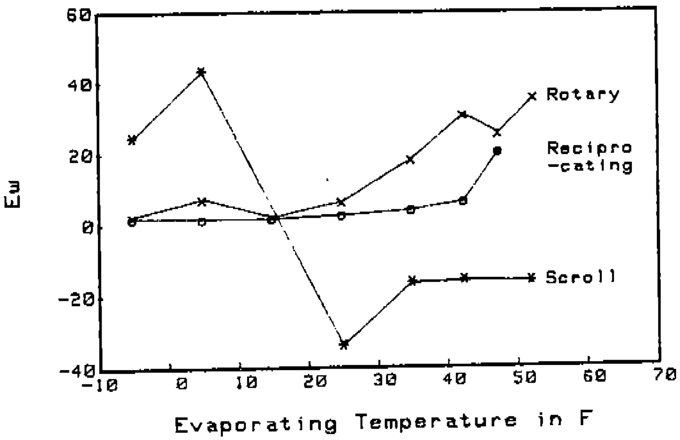
The power elasticity of EER is quite elastic as shown in Fig 2 for given (reciprocating) type of compressor. It becomes even more elastic at high Te and low condensing temperature (Tc). This means that EER becomes very responsive (elastic) under high mass flow condition. On the other hand, the coefficients approach to a certain value (1.5-2 in this case) at low Te regardless of Tc. In other words, percentage changes in input power and EER stay at a fixed ratio at low Te.

Fig.2 Power Elasticity (Ew) of Energy Efficiency Ratio



To illustrate differences in the capacity elasticity, three types of compressors, rotary, reciprocating, and scroll, are compared. Figure 3 shows the elasticity values as a function of Te at a fixed Tc. The elasticity coefficient is quite elastic with increasing trend with Te. Though the one odd-looking curve may be an accuracy problem with data, the importance of the opposite sign remains.

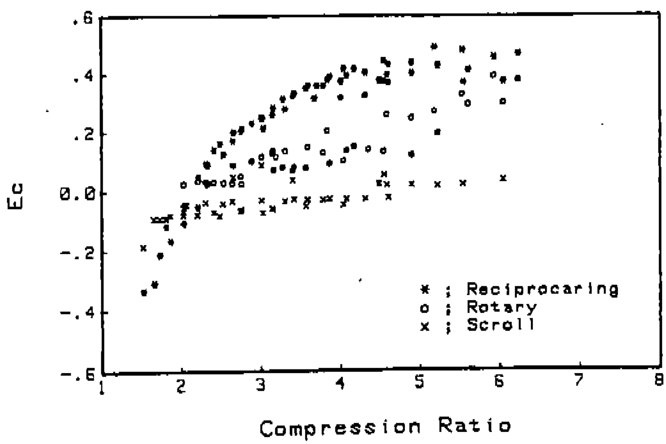
Fig.3 Power Elasticity (Ew) at a Fixed 100 F Condensing Temperature



ELASTICITY AS A FUNCTION OF COMPRESSION RATIO

Compression ratio being an important factor for compressor performance, it will be intuitively interesting to examine an elasticity as a function of compression ratio. Figure 4 plots the calculated capacity elasticity, Ec as a function of compression ratio. The curves all show positive slopes, but each type of compressor distinguishes itself from the other two.

Fig.4 Capacity Elasticity (Ec) as a Function of Compression Ratio



FACTORS AFFECTING MAGNITUDE OF ELASTICITY

Time perspective - Time perspective influences elasticity. Consider that initial break-in run results in reduction in input power during early break-in run. The time span could range from a few hours to weeks, somewhat depending on the running condition of the compressors.

Type of Compressor - The type of compressor does differentiate the magnitude of elasticity and direction of changes in some cases.

Operating Condition - The operating condition of the compressor affects both magnitude and rate of change in elasticity as shown in Figure 1.

Compression Ratio - Figure 4 illustrates the effect of compression ratio on elasticity.

CONCLUSIONS

The economic concept of elasticity can be applied to compressor performance variables. Out of the 20 elasticity coefficients possible from five performance variables capacity, input power, mass flow rate, energy efficiency ratio, and electrical current, one half of them are inverse relationship with respect to the other half. Given an elasticity coefficient, the value varies widely over the operating range of compressor. Though the magnitude of the computed values is roughly similar, the types of compressors show different characteristics with elasticity coefficients.

Capacity elasticity of input power is inelastic over a wide operating range. The elasticity approaches zero at an evaporating temperature of near 50°F (10°C) for the case examined. The elasticity coefficient curves as a function of compression ratio are all positively sloped, but each different type of compressor distinguishes itself from the other two. On the other hand, the power elasticity of EER becomes very elastic at high evaporating and low condensing temperatures. That means that EER becomes very responsive (elastic) under high mass flow condition. Compressor run time, type of compressor, operating condition, and compression ratio are factors which effect elasticity coefficients.